Universal Neutrino Mass Hierarchy and Cosmological Baryon Number Asymmetry

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Abstract

We conjecture that three light Majorana neutrinos and their right-handed counterparts may have a universal geometric mass hierarchy. Incorporating this phenomenological conjecture with the Fritzsch texture of lepton mass matrices in a simple seesaw mechanism, we show that it is possible to simultaneously account for current neutrino oscillation data and the cosmological baryon number asymmetry via leptogenesis.

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Recent solar [1], atmospheric [2], reactor (KamLAND [3] and CHOOZ [4]) and accelerator (K2K [5]) neutrino oscillation experiments provide us with very compelling evidence that neutrinos have nonvanishing masses and their mixing involves two large angles ($\theta_{12} \sim 33^{\circ}$ and $\theta_{23} \sim 45^{\circ}$) and one small angle ($\theta_{13} < 13^{\circ}$). These important results imply that the minimal standard electroweak model, in which neutrinos are massless Weyl particles, is actually incomplete. A very simple extension of the standard model is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under SU(2)_L × U(1)_Y gauge transformation. In this case, the Lagrangian relevant for lepton masses can be written as

$$-\mathcal{L}_{\text{lepton}} = \overline{l}_{\text{L}} \tilde{\phi} Y_l E + \overline{l}_{\text{L}} \phi Y_{\nu} N + \frac{1}{2} \overline{N^{\text{c}}} M_{\text{R}} N + \text{h.c.} , \qquad (1)$$

where $l_{\rm L}$ denotes the left-handed lepton doublet, E and N stand respectively for the right-handed charged lepton and Majorana neutrino singlets, and ϕ is the Higgs-boson weak isodoublet. After spontaneous gauge symmetry breaking, we obtain the charged lepton mass matrix $M_l \equiv Y_l \langle \phi \rangle$ and the Dirac neutrino mass matrix $M_{\rm D} \equiv Y_\nu \langle \phi \rangle$ with $\langle \phi \rangle \approx 174$ GeV. The scale of $M_{\rm R}$ may be considerably higher than $\langle \phi \rangle$, because right-handed neutrinos are SU(2)_L singlets and their mass term is not subject to electroweak symmetry breaking. As a consequence, the effective (light and left-handed) neutrino mass matrix M_ν can be derived from $M_{\rm D}$ and $M_{\rm R}$ via the well-known seesaw mechanism [6]: $M_\nu \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$. It becomes clear that the smallness of left-handed neutrino masses is attributed to the largeness of right-handed neutrino masses. The phenomenon of lepton flavor mixing, which has shown up in both solar and atmospheric neutrino oscillations, arises from a nontrivial mismatch between the diagonalizations of M_l and M_ν .

Note that lepton number violation induced by the third term of $\mathcal{L}_{\text{lepton}}$ allows decays of the heavy Majorana neutrinos N_i (for i=1,2,3) to happen. Because the decay can occur at both tree and one-loop levels, their interference may lead to a CP-violating asymmetry ε_i between the CP-conjugated $N_i \to l + \phi^{\dagger}$ and $N_i \to l^c + \phi$ processes [7]. If the masses of N_i are hierarchical (i.e., $M_1 < M_2 < M_3$), the interactions of N_1 can be in thermal equilibrium when N_2 and N_3 decay. The asymmetries ε_2 and ε_3 are therefore erased before N_1 decays, and only the asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives. In the flavor basis where M_R is diagonal and positive, we have [8]

$$\varepsilon_1 \approx -\frac{3M_1}{16\pi} \left[\frac{\operatorname{Im} \left[(\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu})_{12} \right]^2}{M_2 (\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu})_{11}} + \frac{\operatorname{Im} \left[(\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu})_{13} \right]^2}{M_3 (\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu})_{11}} \right] , \qquad (2)$$

where $\mathbf{Y}_{\nu} \equiv Y_{\nu}U_{\mathrm{R}}$ with U_{R} being a unitary matrix defined to diagonalize M_{R} (namely, $U_{\mathrm{R}}^T M_{\mathrm{R}} U_{\mathrm{R}} = \mathrm{Diag}\{M_1, M_2, M_3\}$), and the condition $M_1^2 \ll M_2^2 \ll M_3^2$ has been taken. The point of leptogenesis is that ε_1 may result in a net lepton number asymmetry $Y_{\mathrm{L}} \equiv (n_{\mathrm{L}} - n_{\bar{\mathrm{L}}})/\mathbf{s} = d\varepsilon_1/g_*$, where \mathbf{s} denotes the entropy of the early universe, $g_* = 106.75$ is an effective number characterizing the relativistic degrees of freedom which contribute to \mathbf{s} , and d measures the dilution effects induced by the lepton-number-violating wash-out processes. This lepton number asymmetry is eventually converted into a net baryon number asymmetry Y_{B} via nonperturbative sphaleron processes [9]: $Y_{\mathrm{B}} \equiv (n_{\mathrm{B}} - n_{\bar{\mathrm{B}}})/\mathbf{s} \approx -0.55 Y_{\mathrm{L}}$. Such an

elegant mechanism may therefore interpret the cosmological matter-antimatter asymmetry, $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$, which is drawn from the recent WMAP observational data [10].

It is a highly nontrivial task to simultaneously account for the cosmological baryon number asymmetry and current neutrino oscillation data, because the textures of Y_{ν} (or $M_{\rm D}$), $M_{\rm R}$ and M_l are completely unknown. Although a number of ansätze for lepton mass matrices have been discussed in the literature [11], most of them remain rather preliminary and have little predictability.

The purpose of this paper is to propose a novel and viable phenomenological model of lepton mass matrices, from which both the cosmological baryon number asymmetry and neutrino mixing parameters can be calculated. Our main conjecture is that three light Majorana neutrinos and their heavy right-handed counterparts have a universal geometric mass hierarchy. Incorporating this conjecture with the Fritzsch texture of lepton mass matrices in a simple seesaw scenario, one may easily account for current neutrino oscillation data at the 3σ level. The leptogenesis mechanism allows us to obtain a very instructive result for the cosmological baryon number asymmetry $Y_{\rm B}$, which depends on a single CP-violating phase. We find that the observed value of $Y_{\rm B}$ can set a strong constraint on the mass scale of three right-handed Majorana neutrinos.

First of all, we conjecture that three light (left-handed) neutrinos have a geometric mass hierarchy at low energies: $m_1/m_2=m_2/m_3\equiv r$. Because of $m_1< m_2$ or r<1 [1], this geometric mass relation cannot be reconciled with the inverted neutrino mass hierarchy (i.e., $m_3< m_1< m_2$). Given the neutrino mass-squared differences $\Delta m_{21}^2\equiv m_2^2-m_1^2=(5.4-9.5)\times 10^{-5}~{\rm eV}^2$ and $\Delta m_{31}^2\equiv m_3^2-m_1^2=(1.4-3.7)\times 10^{-3}~{\rm eV}^2$, which have been determined from a global analysis of current neutrino oscillation data [12], it is easy to obtain

$$\begin{split} m_1 &= \frac{r^2}{\sqrt{1-r^4}} \sqrt{\Delta m_{31}^2} \ , \\ m_2 &= \frac{r}{\sqrt{1-r^4}} \sqrt{\Delta m_{31}^2} \ , \\ m_3 &= \frac{1}{\sqrt{1-r^4}} \sqrt{\Delta m_{31}^2} \ , \end{split} \tag{3}$$

where $r = \sqrt{\Delta m_{21}^2/(\Delta m_{31}^2 - \Delta m_{21}^2)}$ lies in the range $0.122 \lesssim r \lesssim 0.270$. A complete determination of the neutrino mass spectrum is therefore available. We stress that the geometric mass hierarchy of three light neutrinos can essentially be extrapolated up to the lowest mass scale of three heavy (right-handed) Majorana neutrinos. Taking the normal mass hierarchy $M_1 < M_2 < M_3$, one may establish a simple relation for m_i (with i = 1, 2, 3) between the electroweak scale $\mu = M_Z$ and the seesaw mass scale $\mu = M_1$ [13]:

$$m_i(M_1) \approx m_i(M_Z)\mathcal{I}_{\alpha} ,$$
 (4)

where $\mathcal{I}_{\alpha} = \exp\left[\int_{\ln M_Z}^{\ln M_1} \alpha(\tau) d\tau\right]$ with $\alpha \approx (\lambda_H - 3g_2^2 + 6f_t^2)/(16\pi^2)$ measures the overall renormalization-group effect of the Higgs self-coupling, gauge coupling and top-quark Yukawa coupling from M_Z to M_1 . Setting $\mathcal{I}_{\alpha} = 1$ at M_Z , we shall get $\mathcal{I}_{\alpha} > 1$ at any energy scales higher than M_Z . Because Eq. (4) is universally valid for three light neutrinos, we

arrive at the geometric hierarchy $m_1(M_1)/m_2(M_1) \approx m_2(M_1)/m_3(M_1)$ to a good degree of accuracy at the seesaw scale.

We further conjecture that three heavy right-handed neutrinos have the same geometric mass hierarchy as three light neutrinos do: $M_1/M_2 = M_2/M_3 = r$. As a consequence of $r \sim 0.2$, the condition $M_1^2 \ll M_2^2 \ll M_3^2$ holds to guarantee the validity of Eq. (2). One may then take advantage of baryogenesis via leptogenesis to fix the seesaw mass scale M_1 (or M_3). To do so, a specific texture of the Dirac neutrino coupling matrix Y_{ν} must be assumed. We find that it is natural to assume three eigenvalues of Y_{ν} , denoted as D_i (for i=1,2,3), to have the same geometric hierarchy as three heavy Majorana neutrinos do: $D_1/D_2 = D_2/D_3 = r$. Now the question is how to incorporate such a common geometric relation into the formula $M_{\nu} \approx Y_{\nu} M_{\rm R}^{-1} Y_{\nu}^T \langle \phi \rangle^2$ at the seesaw scale ¹.

We consider a simple ansatz of lepton mass matrices to accommodate the conjectures made above. At the scale $\mu = M_1$, the textures of Y_{ν} and $M_{\rm R}$ are required to have a universal Fritzsch form: $Y_{\nu} = D_3 F_{\rm D}$ and $M_{\rm R} = M_3 F_{\rm R}$ with [15]

$$F_{\lambda} = \begin{pmatrix} \mathbf{0} & ce^{i\varphi_{\lambda}} & \mathbf{0} \\ ce^{i\varphi_{\lambda}} & \mathbf{0} & be^{i\phi_{\lambda}} \\ \mathbf{0} & be^{i\phi_{\lambda}} & a \end{pmatrix} , \tag{5}$$

where $\lambda=\mathrm{D}$ or $\mathrm{R},\,a=1-r+r^2,\,b=(1-r)\sqrt{r(1+r^2)/a}$ and $c=r\sqrt{r/a}$. It is easy to check that three eigenvalues of F_λ have the geometric hierarchy $r^2:r:1$. The effective neutrino mass matrix M_ν at low energies may also take the Fritzsch texture: $M_\nu(M_Z)=m_3F_\nu$, where F_ν has the same form as F_λ defined in Eq. (5) but its two phases (denoted by ϕ_ν and φ_ν) are free parameters. Since the running effects of three lepton flavor mixing angles and three CP-violating phases are negligibly small in the standard model with a normal neutrino mass hierarchy 2 , we extrapolate $M_\nu(M_Z)$ up to the scale $\mu=M_1$ in a good approximation: $M_\nu(M_1)\approx m_3\mathcal{I}_\alpha F_\nu$, where $\mathcal{I}_\alpha\sim 1.4$ for $M_1\sim 10^{10}$ GeV [13]. It is then straightforward to obtain the seesaw relation $M_\nu(M_1)\approx Y_\nu M_\mathrm{R}^{-1}Y_\nu^{-1}\langle\phi\rangle^2$, only if the phase condition $\phi_\mathrm{D}-\phi_\mathrm{R}=\varphi_\mathrm{D}-\varphi_\mathrm{R}$ is satisfied. From this seesaw formula, $D_3=\sqrt{m_3M_3\mathcal{I}_\alpha/\langle\phi\rangle}$, $\phi_\nu=2\phi_\mathrm{D}-\phi_\mathrm{R}$ and $\varphi_\nu=2\varphi_\mathrm{D}-\varphi_\mathrm{R}$ can be derived. Our result shows that the Fritzsch texture must be seesaw-invariant, if the eigenvalues of Y_ν and M_R share a universal geometric hierarchy and their phase parameters satisfy a very simple relation. Indeed,

$$\phi_{\nu} - \varphi_{\nu} = \phi_{D} - \varphi_{D} = \phi_{R} - \varphi_{R} \tag{6}$$

holds in this phenomenological scenario. We argue that the texture zeros of F_{λ} could stem from an underlying horizontal flavor symmetry [16]. Although the dynamical reason for

¹Note that we have omitted possible seesaw threshold effects [14] at this point. Such effects are expected to be insignificant and even negligible in our phenomenological scenario, because there is only a span of less than two orders of magnitude from M_3 to M_1 (namely, $M_1/M_3 = r^2 \sim 0.04$).

²This point is also true in the minimal supersymmetric standard model, only if the value of $\tan \beta$ is not extremely large [13].

a universal geometric mass hierarchy of light and heavy Majorana neutrinos is completely unknown, we do have observed the approximate geometric hierarchy of up- and down-quark masses (i.e., $m_u/m_c \sim m_c/m_t$ and $m_d/m_s \sim m_s/m_b$ [17]).

We proceed to take the charged-lepton Yukawa coupling matrix Y_l to have a generic Fritzsch texture with no geometric mass hierarchy:

$$Y_{l} = \begin{pmatrix} \mathbf{0} & \tilde{c}e^{i\varphi_{l}} & \mathbf{0} \\ \tilde{c}e^{i\varphi_{l}} & \mathbf{0} & \tilde{b}e^{i\phi_{l}} \\ \mathbf{0} & \tilde{b}e^{i\phi_{l}} & \tilde{a} \end{pmatrix}, \tag{7}$$

where $\tilde{a} \approx m_{\tau}/\langle \phi \rangle$, $\tilde{b} \approx \sqrt{m_{\mu}m_{\tau}}/\langle \phi \rangle$ and $\tilde{c} \approx \sqrt{m_{e}m_{\mu}}/\langle \phi \rangle$. Because of $m_{e}/m_{\mu} \sim 0.005$ and $m_{\mu}/m_{\tau} \sim 0.06$ [17], the contribution of Y_{l} (or M_{l}) to lepton flavor mixing is not expected to be very significant. A careful numerical analysis made in Ref. [18] has shown that the Fritzsch texture of lepton mass matrices is compatible with current neutrino oscillation data at the 3σ level. We find that the same conclusion can be drawn when the geometric mass hierarchy of three light Majorana neutrinos is taken. In this case, the allowed values of r are restricted to the range $0.215 \lesssim r \lesssim 0.270$.

Now let us turn to the cosmological baryon number asymmetry. To calculate the CP-violating asymmetry ε_1 in Eq. (2), we need to figure out the unitary matrix U_R which is defined to diagonalize M_R . The result is

$$U_{\rm R} = \frac{1}{(1+r)\sqrt{a}} \begin{pmatrix} e^{i(\phi_{\rm R} - \varphi_{\rm R})} & 0 & 0\\ 0 & e^{-i\phi_{\rm R}} & 0\\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -i\sqrt{r(1+r^2)} & r^2\\ \sqrt{ra} & i\sqrt{(1+r^2)a} & \sqrt{ra}\\ -r\sqrt{1+r^2} & -i(1-r)\sqrt{r} & \sqrt{1+r^2} \end{pmatrix}, (8)$$

where $a = 1 - r + r^2$ has been given below Eq. (5). After a straightforward calculation, we arrive at

$$\varepsilon_1 = -\frac{3M_1 m_3 \mathcal{I}_{\alpha} (1 - r)^2 (1 + r^2) \sin^3 \omega \cos \omega}{2\pi \langle \phi \rangle^2 \left[r^2 (1 + r)^2 + 4(1 - r)(1 + r^2) \sin^2 \omega \right]},$$
(9)

where $\omega \equiv (\phi_{\rm D} - \phi_{\rm R})/2$ and m_3 has been given in Eq. (3). Note that ε_1 does not depend on the phase difference $(\varphi_{\rm D} - \varphi_{\rm R})$, because the latter is cancelled out in $(\mathbf{Y}_{\nu}^{\dagger}\mathbf{Y}_{\nu})$ due to the phase relation in Eq. (6). At the seesaw scale, the effective neutrino mass parameter $\tilde{m}_1 \equiv (\mathbf{Y}_{\nu}^{\dagger}\mathbf{Y}_{\nu})_{11}\langle\phi\rangle^2/M_1$ explicitly reads

$$\tilde{m}_1 = m_1 \mathcal{I}_\alpha \left[1 + \frac{4(1-r)(1+r^2)\sin^2 \omega}{r^2(1+r)^2} \right] , \qquad (10)$$

where m_1 has been given in Eq. (3). If \tilde{m}_1 lies in the range 10^{-2} eV $\lesssim \tilde{m}_1 \lesssim 1$ eV (the so-called strong washout regime [19]), one may estimate the dilution factor d by using the approximate formula $d \approx 0.02 \times (0.01 \text{ eV}/\tilde{m}_1)^{1.1}$ given in Ref. [19] or

$$\frac{1}{d} \approx \frac{3.3 \times 10^{-3} \text{ eV}}{\tilde{m}_1} + \left(\frac{\tilde{m}_1}{5.5 \times 10^{-4} \text{ eV}}\right)^{1.16} \tag{11}$$

presented in Ref. [20]. These two simple power laws lead respectively to $d \approx 2.0 \times 10^{-2}$ and 3.4×10^{-2} at $\tilde{m}_1 \approx 0.01$ eV, or $d \approx 1.3 \times 10^{-4}$ and 1.6×10^{-4} at $\tilde{m}_1 \approx 1$ eV. Therefore their

difference is rather insignificant, less than a factor of 2 in the chosen region of \tilde{m}_1 . Given appropriate inputs of ω and M_1 , the observed baryon number asymmetry $Y_{\rm B}$ can then be reproduced from this simple leptogenesis scenario.

For the purpose of illustration, we typically take $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$, r = 0.25 and $m_H = 144 \text{ GeV}$ (Higgs masss) to evaluate $Y_{\rm B}$. Restricting the output of $Y_{\rm B}$ to the generous range $7 \times 10^{-11} \lesssim Y_{\rm B} \lesssim 10^{-10}$ [10], we arrive at the allowed region of ω and M_1 as shown in FIG. 1. Note that regions (A) and (B) are obtained by using the approximate formulas of d given in Refs. [19] and [20], respectively. One can see that the lower bound of M_1 is about 3.0×10^{10} GeV for region (A) or 1.7×10^{10} GeV for region (B). In both cases, the lower limit of ω is about 11.4°. When M_1 is much larger than 10^{12} GeV, the value of ω approaches 90°. This result is a straightfoward consequence of $Y_{\rm B} \propto \varepsilon_1 \propto M_1 \sin^3 \omega \cos \omega$. We have noticed that our numerical outputs are insensitive to the Higgs mass m_H , which slightly affects the running function \mathcal{I}_{α} . The effective neutrino mass \tilde{m}_1 is found to monotonically increase with ω , from 0.01 eV to 0.16 eV in the interval $\omega \in [11.4^{\circ}, 90^{\circ})$. Hence the lepton-number-violating wash-out processes (measured by d) are quite efficient in our scenario.

It is worth mentioning that the above calculation of Y_B can simply be extended to the minimal supersymmetric standard model. The sizes of ε_1 and g_* in the supersymmetric case are twice as large as in the standard model [8], thus the two effects tend to cancel in the estimate of Y_L . For mild values of $\tan \beta$ (from 10 to 50, for example), the supersymmetric running function \mathcal{I}_{α} is essentially comparable in magnitude with its standard model counterpart [13]. Therefore, the numerical result shown in FIG. 1 is roughly valid for the minimal supersymmetric standard model.

It is also worth remarking that the CP-violating asymmetry ε_1 has no direct connection with the low-energy CP violation in neutrino oscillations. The reason is simply that the former is governed by the phase difference $(\phi_{\rm D}-\phi_{\rm R})$, while the latter is associated with the phase differences $(\phi_{\nu}-\phi_l)$ and $(\varphi_{\nu}-\varphi_l)$. This feature is certainly dependent on the phase choice of lepton mass matrices. If Y_l in Eq. (7) is taken to be real (i.e., $\phi_l=\varphi_l=0$ or π), for instance, CP violation at low energies turn out to rely on the phase parameters $\phi_{\nu}=2\phi_{\rm D}-\phi_{\rm R}$ and $\varphi_{\nu}=2\varphi_{\rm D}-\varphi_{\rm R}$. In this case, there appears an indirect connection between CP violation at high scales and that at low scales.

In summary, we have proposed a simple but viable seesaw model of lepton mass matrices based on the phenomenological conjecture that left- and right-handed Majorana neutrinos have a universal geometric mass hierarchy. The cosmological baryon number asymmetry and current neutrino oscillation data can simultaneously be interpreted in this model with very few free parameters. A test of our conjecture and the model itself is possible at low energies, provided more accurate neutrino data are accumulated in the near future.

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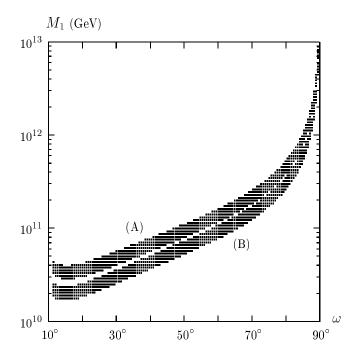


FIG. 1. The allowed ranges of ω and M_1 to reproduce $7 \times 10^{-11} \le Y_{\rm B} \le 10^{-10}$ via leptogenesis, where $\Delta m_{31}^2 = 2.5 \times 10^{-3} \ {\rm eV^2}, \ r = 0.25$ and $m_H = 144$ GeV have typically been input. Note that regions (A) and (B) are obtained, respectively, by using the approximate formulas of d given in Refs. [19] and [20].